DYNAMICAL GENERATION OF TOPOLOGICALLY MASSIVE THREE-DIMENSIONAL GAUGE FIELDS AND COMPOSITE FERMIONS

E.R. NISSIMOV and S.J. PACHEVA
Institute of Nuclear Research and Nuclear Energy, Sofia 1184, Bulgaria

ABSTRACT. Phase transition, non-perturbative particle spectra including fermion-boson bound states and dynamical generation of topological gauge-invariant mass terms for the gauge fields in the general class of three-dimensional Higgs models with fermions are derived within the $1/N$ expansion.

1. Recently much attention was devoted to a new *gauge-invariant* mechanism for generation of masses for the gauge fields [1, 2] as an alternative of the standard Higgs mechanism. In Reference [2] the novel non-trivial properties of $D = 3$ quantum chromodynamics and $D = 3$ Einstein gravity (D being the space-time dimension) with *explicitly added* topological gauge-invariant mass terms (TGIMTs) were studied in great detail. Since TGIMTs break P- and T-invariance (space- and time-reflection) it was suggested (and explicitly shown in the leading order of the coupling constant perturbation theory) [2] that TGIMTs can be dynamically generated (i.e., can arise through quantum effects) if the initial classical theory contains P- and T-breaking terms (e.g., non-zero mass terms for fermion fields).

In the present note we consider the general class of $D = 3$ Higgs models with fermions (HMF_3), possessing internal $U(N) \times U(n)_{gauge}$ symmetry ($n < N$), within the $1/N$ expansion, which exhibit a richer structure. Their phase transition and non-perturbative particle spectra in the corresponding phases including bound states of bosons and fermions and dynamically generated TGIMTs (Equations (11c), (15) below) are derived. It is shown that P- and *T-non-invariants* of the classical theory in this more complicated case is, unlike [2], neither necessary nor sufficient for dynamical generation of TGIMTs. More precisely:

(a) The P- and T-invariant generalized non-linear sigma models with fermions (Equation (3) below) which are particular cases of HMF_3 exhibit dynamical P, T-breakdown within the $1/N$ expansion through a dynamical generation of fermion mass m_F (Equation (8) below), the latter giving rise to TGIMTs (Equation (15) below). This result was already derived in the special case of $D = 3$ supersymmetric generalized non-linear sigma models [3] (Equation (3′) below).

(b) The pre-scaling critical theory of the general HMF_3 (Equation (14) below) is P, T-non-invariant, however, no TGIMTs do arise there since the dynamical fermion mass vanishes.

2. The Lagrangian of HMF_3 reads:

Letters in Mathematical Physics **6** (1982) 361–371. 0377–9017/82/0065–0361 $01.10.

$$\mathcal{L}_{\mathrm{HMF}_3} = \mathcal{L}_\varphi + \mathcal{L}_\psi + Nn\mathcal{L}_A + \mathcal{L}_C;$$

$$\mathcal{L}_\varphi = |\nabla_\mu\varphi|^2 - M_B^2\varphi^*\varphi - \frac{\lambda_1\mu}{2Nn}(\varphi^*\varphi)^2 - \frac{\lambda_3\mu}{2Nn}(\varphi^*\tau_A\varphi)^2 - \frac{\lambda_2}{3(Nn)^2}(\varphi^*\varphi)^3 -$$

$$- \frac{\lambda_4}{3(Nn)^2} d_{ABC}(\varphi^*\tau_A\varphi)(\varphi^*\tau_B\varphi)(\varphi^*\tau_C\varphi) - \frac{\lambda_5}{2(Nn)^2}(\varphi^*\varphi)(\varphi^*\tau_A\varphi)^2;$$

$$\mathcal{L}_\psi = \frac{i}{2}\overline{\psi}\overleftrightarrow{\nabla\!\!\!/}\psi - M_F\overline{\psi}\psi + \frac{g_1}{Nn}(\overline{\psi}\psi)(\varphi^*\varphi) + \frac{f_1}{Nn}(\overline{\psi}\tau_A\psi)(\varphi^*\tau_A\varphi) + \frac{g_3}{Nn}(\overline{\psi}\varphi)(\varphi^*\psi) +$$

$$+ \frac{f_3}{Nn}(\overline{\psi}\tau_A\varphi)(\varphi^*\tau_A\psi) + \frac{g_2}{2Nn}[(\overline{\psi}\varphi)\mathscr{C}(\overline{\psi}\varphi) + (\varphi^*\psi)\mathscr{C}(\varphi^*\psi)] + \qquad (1)$$

$$+ \frac{f_2}{2Nn}[(\overline{\psi}\tau_A\varphi)\mathscr{C}(\overline{\psi}\tau_A\varphi) + (\varphi^*\tau_A\psi)\mathscr{C}(\varphi^*\tau_A\psi)];$$

$$\mathcal{L}_A = -\frac{1}{4e_1^2\mu}F_{\kappa\lambda}^2(A_{(0)}) - \frac{1}{4ne_3^2\mu}\operatorname{tr}(\underset{\sim}{F}_{\kappa\lambda}^2(\underset{\sim}{A})) - B_{(0)}\partial_\mu A^\mu_{(0)} - \frac{1}{n}\operatorname{tr}(\underset{\sim}{B}\partial_\mu \underset{\sim}{A}^\mu) +$$

$$+ (Nn)^{-1}\operatorname{tr}(\underset{\sim}{\chi}^*\partial^\mu\nabla_\mu\underset{\sim}{\chi});$$

$$\mathcal{L}_C = \frac{Nn}{8e_2^2\mu}\overline{C}_{(0)}i\partial\!\!\!/ C_{(0)} + \frac{N}{8e_4^2\mu}\operatorname{tr}(\overline{\underset{\sim}{C}}i\nabla\!\!\!/\underset{\sim}{C}) + \frac{i}{2}\overline{\psi}(C_{(0)} + \underset{\sim}{C})\varphi - \frac{i}{2}\varphi^*(\overline{C}_{(0)} + \overline{\underset{\sim}{C}})\psi;$$

with the following notations:

$$F^{\mu\nu}(A_{(0)}) = \partial^\mu A^\nu_{(0)} - \partial^\nu A^\mu_{(0)}; \qquad \underset{\sim}{F}^{\mu\nu}(\underset{\sim}{A}) = \partial^\mu\underset{\sim}{A}^\nu - \partial^\nu\underset{\sim}{A}^\mu + i[\underset{\sim}{A}^\mu, \underset{\sim}{A}^\nu],$$

$$(\nabla_\mu\varphi)^k_a = \partial_\mu\varphi^k_a + iA_{(0)\mu}\varphi^k_a + i\underset{\sim}{A}^{kl}_\mu\varphi^l_a \quad \text{(the same for } (\nabla_\mu\psi)^k_a\text{)},$$

$$(\nabla_\mu\underset{\sim}{C})^{kl} = \partial_\mu\underset{\sim}{C}^{kl} + i[\underset{\sim}{A}_\mu, \underset{\sim}{C}]^{kl} \quad \text{(the same for } \nabla_\mu\underset{\sim}{\chi}\text{)},$$

$$O\!\!\!/ \equiv O^\nu\gamma_\nu, \qquad \overline{O} \equiv O^*\gamma_0, \qquad \mathscr{C}^{-1}\gamma_\mu\mathscr{C} = -\gamma_\mu^T, \qquad \mathscr{C}^T = -\mathscr{C} \text{ (charge conjugation matrix)},$$

$$\underset{\sim}{O}^{kl} \equiv O_A\tau_A^{kl}, \qquad \operatorname{tr}(\tau_A\tau_B) = n\delta_{AB}, \qquad \{\tau_A, \tau_B\} = 2(\delta_{AB} + d_{ABC}\tau_C).$$

In (1) μ is an arbitrary mass scale, so that all coupling constants are set dimensionless. The auxiliary fields $B_{(0)}$, $\underset{\sim}{B}$ enforce Landau gauge conditions for the U(1) and SU(n) gauge fields $A^\mu_{(0)}$, $\underset{\sim}{A}^\mu$; $\underset{\sim}{\chi}$ are the corresponding Faddeev–Popov ghost fields. $C_{(0)}$ and $\underset{\sim}{C}$ are Majorana fermionic fields ($\overline{C} = \mathscr{C}^{-1}C$) transforming under the singlet and the adjoint representation of $\mathrm{SU}(n)_{\mathrm{gauge}}$. The hermitian $n \times n$ matrices τ_A, $A = 1, ..., n^2 - 1$, span a hermitian basis of the SU(n) Lie-algebra. Summation over repeated indices ('flavor' ones $a, b = 1, ..., N$; 'color' ones $k, l = 1, ..., n$; adjoint-SU(n) ones $A, B = 1, ..., n^2 - 1$ and Lorentz ones) is understood and the latter will be suppressed for brevity. In particular, on the submanifold M_{SS} of the coupling constant space:

$$g_i = g_{(0)}, f_i = f_{(0)}, \quad i = 1, 2, 3; \qquad \lambda_1 = -4g_{(0)}^2/T, \lambda_2 = 3g_{(0)}^2, \lambda_3 = -g_{(1)}^2/T,$$

$$\lambda_4 = 3g_{(1)}^2, \lambda_5 = 2g_{(1)}(g_{(1)} + 2g_{(0)}); \qquad M_B = M_F = \mu g_{(0)}/T;$$

we obtain (cf. [4]):

$$\mathcal{L}_{\mathrm{HMF}_3}\big|_{M_{SS}} = \mathcal{L}_{\text{supersymmetric Higgs}}^{\star} \tag{1'}$$

To construct the $1/N$ expansion of the quantum HMF_3 we use the standard device to convert the Lagrangian (1) into a quadratic function in φ, ψ by means of introducing a set of auxiliary fields ($\alpha_{(0)}$, $\underset{\sim}{\alpha}$, $\sigma_{(0)}$, $\underset{\sim}{\sigma}$ – real bosonic, $\kappa_{(0)}$, $\underset{\sim}{\kappa}$ – Majorana fermionic, $\rho_{(0)}$, $\underset{\sim}{\rho}$ – complex fermionic):

$$\begin{aligned}
&\mathcal{L}'_{\mathrm{HMF}_3} = \mathcal{L}'_\varphi + \mathcal{L}'_\psi + Nn(\mathcal{L}_A + \mathcal{L}'_C);\\
&\mathcal{L}'_\varphi = |\nabla_\mu\varphi|^2 - \varphi^*(\alpha_{(0)} + \underset{\sim}{\alpha})\varphi + Nn\mathcal{L}_{\alpha\sigma},\\
&\mathcal{L}'_\psi = \frac{i}{2}\overline{\psi}\overset{\leftrightarrow}{\nabla\!\!\!/}\psi + \overline{\psi}(\sigma_{(0)} + \underset{\sim}{\sigma})\psi + \overline{\psi}[\kappa_{(0)} + \underset{\sim}{\kappa} + \rho_{(0)} + \underset{\sim}{\rho} + \frac{i}{2}(C_{(0)} + \underset{\sim}{C})]\varphi +\\
&\qquad + \varphi^*[\overline{\kappa}_{(0)} + \overline{\underset{\sim}{\kappa}} + \overline{\rho}_{(0)} + \overline{\underset{\sim}{\rho}} - \frac{i}{2}(\overline{C}_{(0)} + \overline{\underset{\sim}{C}})]\psi + Nn\mathcal{L}_{\kappa\rho},\\
&\mathcal{L}'_C = (8e_2^2\mu)^{-1}\overline{C}_{(0)}i\partial\!\!\!/ C_{(0)} + (8e_4^2 n\mu)^{-1}\,\mathrm{tr}\,(\overline{\underset{\sim}{C}}i\nabla\!\!\!/\underset{\sim}{C}),\\
&\mathcal{L}_{\alpha\sigma} \equiv \frac{\mu}{T}\alpha_{(0)} + \frac{u_1}{8}\alpha_{(0)}\sigma_{(0)} + \frac{u_5}{8n}\,\mathrm{tr}\,(\underset{\sim}{\alpha}\underset{\sim}{\sigma}) - \mu^2 a_1\sigma_{(0)} - \mu a_2\sigma_{(0)}^2 -\\
&\qquad - \frac{1}{n}\mu a_3\,\mathrm{tr}\,(\underset{\sim}{\sigma}^2) - \frac{u_3}{8}\sigma_{(0)}^3 - \frac{u_7}{8n}\,\mathrm{tr}\,(\underset{\sim}{\sigma}^3) - \frac{u_9}{8n}\sigma_{(0)}\,\mathrm{tr}\,(\underset{\sim}{\sigma}^2),\\
&\mathcal{L}_{\kappa\rho} \equiv -\frac{1}{16}[u_2\overline{\kappa}_{(0)}\kappa_{(0)} + u_6\overline{\kappa}_A\kappa_A + \frac{1}{u_4}\overline{\rho}_{(0)}\rho_{(0)} + \frac{1}{u_8}\overline{\rho}_A\rho_A].
\end{aligned} \tag{2}$$

The new set of coupling constants appearing in (2) is defined through the old one in (1) as:

$$1/T = M_F(\mu g_1)^{-1},$$

$$a_1 = [(M_B/\mu)^2 + \lambda_1/T + \lambda_2/T^2]g_1^{-1}, \qquad a_2 = (\tfrac{1}{2}\lambda_1 + \lambda_2/T)g_1^{-2}, \qquad a_3 = (\lambda_3 + \lambda_5/T)(2f_1^2)^{-1},$$

$$u_1 = 8/g_1, \qquad u_2 = 8/g_2, \qquad u_3 = 8\lambda_2/3g_1^3, \qquad u_4 = \tfrac{1}{16}(g_3 - g_2),$$

★In Reference [4], on the r.h. side of Equation (1') ($\mathcal{L}_{\text{supersymm. Higgs}}$) there is a missing term:

$$-g_{(1)}(g_{(1)} + 2g_{(0)})(Nn)^{-2}(\varphi^*\varphi - Nn\mu/T)(\varphi^*\tau_A\varphi)^2.$$

$$u_5 = 8/f_1, \quad u_6 = 8/f_2, \quad u_7 = 8\lambda_4/3f_1^3, \quad u_8 = \tfrac{1}{16}(f_3 - f_2), \quad u_9 = 4\lambda_5(g_1 f_1^2)^{-1}.$$

The generalized non-linear sigma models with fermions (introduced in [5] for $D = 2$):

$$\mathcal{L}_{\mathrm{GNLSM}+F} = |\nabla_\mu \varphi|^2 + \frac{i}{2}\overline{\psi}\overleftrightarrow{\not{\partial}}\psi + (4Nna_2\mu)^{-1}(\overline{\psi}\psi)^2 + (4Nna_3\mu)^{-1}(\overline{\psi}\tau_A\psi)^2,$$

$$\varphi^*\tau_A\varphi = 0, \qquad \varphi^*\varphi - Nn\mu/T = 0, \qquad \overline{\psi}\varphi = \varphi^*\psi = \overline{\psi}\tau_A\varphi = \varphi^*\tau_A\psi = 0, \tag{3}$$

$$A_{(0)\nu} = (2Nn\mu/T)^{-1}[i\varphi^*\overleftrightarrow{\partial}_\nu\varphi + \overline{\psi}\gamma_\nu\psi], \qquad A_{B\nu} = (2Nn\mu/T)^{-1}[i\varphi^*\tau_B\overleftrightarrow{\partial}_\nu\varphi + \overline{\psi}\gamma_\nu\tau_B\psi],$$

are special limiting cases of (2) when:

$$a_1 = 0, \qquad u_i = 0 \quad (i = 1, \ldots, 9), \qquad e_j^2 \to \infty \quad (j = 1, \ldots, 4).$$

Furthermore (cf. [3]):

$$\mathcal{L}_{\mathrm{GNLSM}+F}\big|_{a_2 = a_3 = 1/T} = \mathcal{L}_{\text{supersymmetric GNLSM}}. \tag{3'}$$

Under P- and T-reflection the fields in (1)–(3) transform as:

$$\varphi^{(P,T)}_{(x)} = \eta_{P,T}\varphi(x_{P,T}); \qquad \psi^{(P,T)}(x) = -i\eta_{P,T}\gamma_{1,2}\psi(x_{P,T}); \qquad C^{(P,T)}(x) = i\gamma_{1,2}C(x_{P,T});$$

$$A_\mu^{(P)}(x) = (A_0, -A_1, A_2)(x_P), \qquad A_\mu^{(T)}(x) = (A_0, -A_1, -A_2)(x_T);$$

$$\alpha^{(P,T)}(x) = \alpha(x_{P,T}); \; \sigma^{(P,T)}(x) = -\sigma(x_{P,T}); \; \kappa^{(P,T)}(x) = i\gamma_{1,2}\kappa(x_{P,T}); \tag{4}$$

$$\rho^{(P,T)}(x) = i\gamma_{1,2}\rho(x_{P,T});$$

$$x_P^\mu \equiv (x^0, -x^1, x^2), \qquad x_T^\mu \equiv (-x^0, x^1, x^2), \qquad \eta_{P,T} = \pm 1.$$

Thus, the models (3), (3′) are P- and T-invariant unlike (1), (1′).

3. Following the standard techniques (see e.g. [6]) the fields $\psi, \varphi_\perp$, where $\varphi = N^{1/2}\varphi_\parallel + \varphi_\perp$, $\varphi^k_{\perp a_1} = 0, a_1 = 1, \ldots, n, \varphi^k_{\parallel a_2} = 0, a_2 = n + 1, \ldots, N$, are integrated out in the generating functional of the Green's functions of (2):

$$\begin{aligned} Z[\text{sources}] &= \text{const}\int D\varphi_\perp D\psi D\varphi_\parallel D\alpha_{(0)} \ldots D\chi \exp\left\{ i\int \mathrm{d}^3x[\mathcal{L}'_{\mathrm{HMF}_3} + \right. \\ &\left. + \text{source terms}]\right\} = (\text{const})'\int D\varphi_\parallel D\alpha_{(0)} \ldots D\chi \exp\{iNS_1 + iS_2\} \end{aligned} \tag{5}$$

and the $1/N$ expansion is generated through expansion of (5) around the constant saddle points of S_1:

$$\hat{\alpha}_{(0)} \equiv m_B^2, \qquad \hat{\sigma}_{(0)} \equiv -m_F, \qquad \hat{\varphi}_{\parallel} \equiv v$$

all other fields having zero stationary values, where:

$$S_1 \equiv i\left(1-\frac{n}{N}\right) \mathrm{Tr}\ln\Delta_B - i\,\mathrm{Tr}\ln\Delta_F + \int \mathrm{d}^3x\,[-\varphi_\parallel^\dagger\Delta_B\varphi_\parallel + n(\mathcal{L}_{\alpha\sigma}+\mathcal{L}_{\kappa\rho}+\mathcal{L}_A+\mathcal{L}'_C)]\,;$$

$$\Delta_B \equiv \nabla_\mu\nabla^\mu + \alpha_{(0)} + \underset{\sim}{\alpha} + [\bar{\kappa}_{(0)} + \underset{\sim}{\bar{\kappa}} + \bar{\rho}_{(0)} + \underset{\sim}{\bar{\rho}} - \frac{i}{2}(\bar{C}_{(0)} + \underset{\sim}{\bar{C}})]\,\Delta_F^{-1}\,[\kappa_{(0)} + \underset{\sim}{\kappa} + \rho_{(0)} + \underset{\sim}{\rho} + \frac{i}{2}(C_{(0)} + \underset{\sim}{C})]$$

$$\Delta_F \equiv i\not{\nabla} + \sigma_{(0)} + \underset{\sim}{\sigma}. \tag{6}$$

S_2 in (5) contains the corresponding source terms.

The stationary equations for S_1 (6):

$$m_B = 4\pi[\mu(1/T_c - 1/T) + |v|^2 + u_1 m_F/8], \qquad |v|^2\delta^{kl} \equiv v_a^{*k} v_a^l, \quad T_c = 4\pi(1+a_0); \tag{7a}$$

$$\frac{1}{2\pi} - \frac{3}{8}u_3 \quad m_F^2 - 2\mu(1/T_c - a_2)m_F + \frac{1}{8}u_1 m_B^2 - \mu^2 a_1 = 0; \tag{7b}$$

$$m_B v = 0; \tag{7c}$$

where the constant a_0 accounts for the arbitrariness in the renormalization of divergent point-loop graphs (cf. [3]), possess three distinct types of solutions.

(i) 'High-temperature' phase solution ($T > T^*$):

$$v = 0; \qquad m_{B,\pm}^{(H)} = 4\pi\mu(1/T_c - 1/T) + \frac{\pi}{2}u_1 m_{F,\pm}^{(H)};$$

$$m_{F,\pm}^{(H)} = 4\pi\mu(1/T_c - a_2)[E \pm (E^2 + \mathcal{H}^{1/2}],$$

$$E \equiv \{1 - \pi^2 u_1^2(1/T_c - 1/T)[4(1/T_c - a_2)]^{-1}\}\left[2 - \frac{3\pi}{2}u_3 + \frac{1}{8}\pi^3 u_1^3\right]^{-1}, \tag{8a}$$

$$\mathcal{H} \equiv \left[\frac{1}{4\pi}a_1 - \frac{\pi}{2}u_1(1/T_c - 1/T)^2\right]\left[(1/T_c - a_2)^2 \;\; 2 - \frac{3\pi}{2}u_3 + \frac{1}{8}\pi^3 u_1^3\Big)\right]^{-1};$$

(ii) 'Low-temperature' phase solution ($T < T^*$):

$$m_B^{(L)} = 0; \qquad |v|^2 = \mu(1/T - 1/T^*);$$

$$m_{F,\pm}^{(L)} = \frac{4\pi\mu(1/T_c - a_2)}{2(1 - \frac{3}{4}\pi u_3)}\left\{\left[1 \pm \frac{a_1(1 - \frac{3}{4}\pi u_3)}{2\pi(1/T_c - a_2)^2}\right]^{1/2}\right\}; \tag{8b}$$

(iii) $T = T^*$: $m_B^{(*)} = 0$, $v^{(*)} = 0$, $m_F^{(*)} = m_{F,\pm}^{(L)}$, (8c)

where the 'critical temperature' T^* is defined as:

$$1/T^* = 1/T_c + (u_1/8\mu)m_{F,\pm}^{(L)}. \tag{9}$$

As in the case of the supersymmetric generalized non-linear sigma models [3] the degeneracy of the saddle point solutions (8) is removed after examining which values of $m_{B,\pm}$, $m_{F,\pm}$ yield an absolute minima of the quantum effective potential in the large N limit:

$$\frac{1}{n}\mathscr{V}_{\text{eff}}^{N=\infty} = -\frac{1}{6\pi}m_B^3(v) + m_B^2(v)[\mu(1/T_c - 1/T) + |v|^2 + u_1 m_F(v)/8] + \\ + (1/6\pi - u_3/8)m_F^3(v) - \mu(1/T_c - a_2)m_F^2(v) - \mu^2 a_1 m_F(v). \tag{10}$$

The functions $m_B(v)$, $m_F(v)$ are determined by (7a, b). The result is:

$$m_F^{(H,L)} = \begin{cases} m_{F,+}^{(H,L)} & \text{for } 1/T_c - a_2 \geqslant 0, \\ m_{F,-}^{(H,L)} & \text{for } 1/T_c - a_2 < 0. \end{cases}$$

Now, from the quadratic part of S_1 (6) the 'free' propagators ($\langle\cdots\rangle^{(0)}$) of the $1/N$ expansion can be found. We write down the latter in a unified notation (in momentum space) simultaneously suited for (i), (ii), (iii) (in each separate case m_B, m_F, v are to be set equal to their corresponding values (8a, b, c)). For the fundamental fields in (1) we obtain:

$$\langle\varphi_a^k\varphi_b^{*l}\rangle^{(0)} = (-i)\delta_{ab}\delta^{kl}[m_B^2 - p^2]^{-1} + \frac{1}{n}(-p^2)^{-2}\{v_a^k v_b^{*l} Nn\langle\alpha_{(0)}\alpha_{(0)}\rangle^{(0)}(u_1) + \\ + (\tau_A v_a)^k(v_b^*\tau_B)^l Nn\langle\alpha_A\alpha_B\rangle^{(0)}(u_5)\}; \tag{11a}$$

$$\langle\psi_a^k\overline{\psi}_b^l\rangle^{(0)} = (-i)\delta_{ab}\delta^{kl}(\not{p} + m_F)[m_F^2 - p^2]^{-1} + \\ + \frac{1}{n}(-p^2)^{-1}\{v_a^k v_b^{*l} Nn\langle\omega_{(0)}\overline{\omega}_{(0)}\rangle^{(0)}(u_2) + (\tau_A v_a)^k(v_b^*\tau_B)^l Nn\langle\omega_A\overline{\omega}_B\rangle^{(0)}(u_6)\}; \tag{11b}$$

$$\omega_{(0),A} \equiv \kappa_{(0),A} + \frac{i}{2}C_{(0),A} + \rho_{(0),A};$$

$$\langle A_{(0)}^\mu A_{(0)}^\nu\rangle^{(0)}(e_1) = (Nn)^{-1}i[\mathscr{F}^2(e_1) - p^2G^2]^{-1}\left\{\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\mathscr{F}(e_1) + i\epsilon^{\mu\nu\lambda}p_\lambda G\right\}; \tag{11c}$$

$$\mathscr{F}(e_1) \equiv -p^2/e_1^2\mu + |v|^2 + (4m_B^2 - p^2)\tfrac{1}{2}F(p^2; m_B) - (4m_F^2 + p^2)\tfrac{1}{2}F(p^2; m_F) + \frac{1}{4\pi}(m_F - m_B), \\ G \equiv 2m_F F(p^2; m_F) = \frac{1}{4\pi}f(p^2/4m_F^2); \tag{11d}$$

$$F(p^2; m) \equiv (8\pi m)^{-1} f(p^2/4m^2), \qquad f(z) \equiv (2z^{1/2})^{-1} \ln\,[(1+z^{1/2})(1-z^{1/2})^{-1}]; \quad (11e)$$

$$\langle A_B^\mu A_C^\nu \rangle^{(0)}(e_3) = \delta_{BC} \langle A_{(0)}^\mu A_{(0)}^\nu \rangle^{(0)}(e_3). \quad (11f)$$

For generic values of the coupling constants there is non-trivial mixing between κ, C and ρ:

$$\langle \kappa_{(0)} \bar{\kappa}_{(0)} \rangle^{(0)}(u_2, u_4) = i(Nn)^{-1} Q_1^{-1}(1 - 8u_4 \mathscr{R}), \quad (12a)$$

$$\langle C_{(0)} \bar{C}_{(0)} \rangle^{(0)}(u_2, u_4; e_2) = 4i(Nn)^{-1} Q_2^{-1}(1 - 8u_4 \mathscr{R}), \quad (12b)$$

$$\langle \rho_{(0)} \bar{\rho}_{(0)} \rangle^{(0)}(u_2, u_4; e_2) = -8iu_4 (Nn)^{-1} [Q_1^{-1}(\mathscr{R} - \tfrac{1}{8}u_2) + Q_2^{-1}(\mathscr{R} + \not{p}/e_2^2 \mu)], \quad (12c)$$

$$\langle \kappa_{(0)} \bar{C}_{(0)} \rangle^{(0)} = \langle C_{(0)} \bar{\kappa}_{(0)} \rangle^{(0)} = 0, \quad (12d)$$

$$\langle \kappa_{(0)} \bar{\rho}_{(0)} \rangle^{(0)}(u_2, u_4) = \langle \rho_{(0)} \bar{\kappa}_{(0)} \rangle^{(0)}(u_2, u_4) = 8iu_4 (Nn)^{-1} Q_1^{-1} \mathscr{R}, \quad (12e)$$

$$\langle C_{(0)} \bar{\rho}_{(0)} \rangle^{(0)}(u_2, u_4; e_2) = -\langle \rho_{(0)} \bar{C}_{(0)} \rangle^{(0)}(u_2, u_4; e_2) = -16u_4 (Nn)^{-1} Q_2^{-1} \mathscr{R}, \quad (12f)$$

$$\langle \kappa_A \bar{\kappa}_B \rangle^{(0)}(u_6, u_8) = \delta_{AB} \langle \kappa_{(0)} \bar{\kappa}_{(0)} \rangle^{(0)}(u_6, u_8), \quad (12g)$$

$$\langle C_A \bar{C}_B \rangle^{(0)}(u_6, u_8; e_4) = \delta_{AB} \langle C_{(0)} \bar{C}_{(0)} \rangle(u_6, u_8; e_4), \text{ etc.,}$$

$$Q_1 \equiv (1 + u_2 u_4) \mathscr{R} - \tfrac{1}{8}u_2, \qquad Q_2 \equiv (1 - \not{p} 8u_4/\mu e_2^2) \mathscr{R} + \not{p}/e_2^2 \mu,$$

$$\mathscr{R} \equiv (2m_F + \not{p}) F(p^2; \tfrac{1}{2}(m_B + m_F)) - 2|v|^2 \not{p}/p^2.$$

Likewise, there is non-trivial α-σ mixing:

$$\langle \alpha_{(0)} \alpha_{(0)} \rangle^{(0)}(u_1, u_3; a_2) = i(Nn)^{-1} \{F(p^2; m_B) - 2|v|^2/p^2 + (u_1/8)^2 \times \times [(4m_F^2 - p^2)F(p^2; m_F) - 2\mu(1/T_c - a_2) + \frac{m_F}{2\pi}(1 - \tfrac{3}{2}\pi u_3)^{-1}]^{-1}\}^{-1}; \quad (13a)$$

$$\langle \sigma_{(0)} \sigma_{(0)} \rangle^{(0)}(u_1, u_3; a_2) = (-1)(Nn)^{-1} \{(4m_F^2 - p^2)F(p^2; m_F) - 2\mu(1/T_c - a_2) + \frac{m_F}{2\pi}(1 - \tfrac{3}{2}\pi u_3) + + (u_1/8)^2 [F(p^2; m_B) - 2|v|^2/p^2]^{-1}\}^{-1}; \quad (13b)$$

$$\langle \alpha_{(0)} \sigma_{(0)} \rangle^{(0)}(u_1, u_3; a_2) = \langle \sigma_{(0)} \alpha_{(0)} \rangle^{(0)}(u_1, u_3; a_2) = i(Nn)^{-1}(u_1/8)\{[(4m_F^2 - p^2)F(p^2; m_F) - - 2\mu(1/T_c - a_2) + \frac{m_F}{2\pi}(1 - \tfrac{3}{2}\pi u_3)]\,[F(p^2; m_B) - 2|v|^2/p^2] + (u_1/8)^2\}^{-1}; \quad (13c)$$

$$\langle \alpha_A \alpha_B \rangle^{(0)}(u_5, \tfrac{1}{3}u_9; a_3) = \delta_{AB} \langle \alpha_{(0)} \alpha_{(0)} \rangle^{(0)}(u_5, \tfrac{1}{3}u_9; a_3),$$

$$\langle\sigma_A\sigma_B\rangle^{(0)}(u_5, \tfrac{1}{3}u_9; a_3) = \delta_{AB}\langle\sigma_{(0)}\sigma_{(0)}\rangle^{(0)}(u_5, \tfrac{1}{3}u_9; a_3), \tag{13d}$$

$$\langle\sigma_A\sigma_B\rangle^{(0)}(u_5, \tfrac{1}{3}u_9; a_3) = \langle\sigma_A\alpha_B\rangle^{(0)}(u_5, \tfrac{1}{3}u_9; a_3) = \delta_{AB}\langle\alpha_{(0)}\sigma_{(0)}\rangle^{(0)}(u_5, \tfrac{1}{3}u_9; a_3).$$

All ultraviolet divergent graphs arising in the computation of (11) – (13) are renormalized by means of a minimal mass-independent 'soft-mass'-type [7] renormalization scheme [3, 4, 8]. If the Pauli–Villars cut-off procedure is employed instead (cf. [2]), the result for G in (11d) changes:

$$G \to \widetilde{G} \equiv 2m_F F(p^2; m_F) - 1/4\pi = \frac{1}{4\pi}(f(p^2/4m_F^2) - 1).$$

Hence, in the 'high-temperature' phase a massless particle pole of $\langle A^\mu_{(0)}A^\nu_{(0)}\rangle^{(0)}$, $\langle A^\mu_B A^\nu_C\rangle^{(0)}$ is created, as already stated in [3, 4] in the case of the supersymmetric generalized non-linear sigma models and the supersymmetric Higgs models. However, there is an argument which rejects the Pauli–Villars procedure as a legitimate renormalization scheme in our case. Namely, we aim at using a unified renormalization scheme simultaneously suited for (i), (ii), (iii) and, in particular, for describing the critical behaviour of HMF_3 (cf. [3, 4, 8]). In the latter case, at the critical point itself [8]:

$$m_B = m_F = v = 0, \qquad a_2 = a_3 = 1/T_c \qquad \text{(pre-scaling critical theory)}$$

$$u_i = 0 \quad (i = 1, \ldots, 9), \qquad e_j^2 = \infty \quad (j = 1, \ldots, 4) \quad \text{(fixed point of the renormalization group [8])} \tag{14}$$

the universal critical theory of HMF_3 (2) (which turns out to be the supersymmetric generalized non-linear sigma model (3′)) acquires P and T-invariance and these cannot be respected by Pauli–Villars cut-offs.

4. Now, from (8), (11)–(13) the properties of the corresponding phases of HMF_3 (within the $1/N$ expansion) can be easily derived.

The 'low-temperature' phase is characterized by spontaneous breaking of the internal $U(N) \times U(n)_{gauge}$ symmetry down to the residual $U(N-n) \times \mathrm{diag}(U(n)_{global} \times U(n)_{gauge})$ symmetry (the latter being the stability subgroup of the 'low-temperature' phase vacuum: $\langle\varphi^k_{\parallel a}\rangle = N^{1/2}(v^k_a + O(N^{-1}))$, and its particle spectrum consists of *only* $n(N-1)$ massless Goldstone bosons and of $n(N-1)$ (massive) fermions corresponding to $\varphi_\perp$, $\psi_\perp$ and $v^{*l}_a\varphi^k_a$, $v^{*l}_a\psi^k_a$ $(k \neq l)$, respectively. All the remaining fields are here 'confined' and, in particular, the Higgs mechanism is suppressed.

The 'high-temperature' phase is $U(N) \times U(n)_{gauge}$ symmetric and its particle spectrum consists of $n \cdot N$ pairs of bosons and fermions (φ^k_a, ψ^k_a-quanta) with (dynamically-generated in the case of (3), (3′)) masses m_B, m_F (8a) and of the following set of dynamically-generated (bound) states:

(α) Massive gauge bosons ($A^\mu_{(0)}$, A^μ_B) due to the dynamically generated TGIMTs in (11c):

$$-(Nn)^{-1}\epsilon_{\mu\nu\lambda}p^{\lambda}\frac{1}{4\pi}f(p^2/4m_F^2)[\mathscr{F}^2(e_{1,3})-p^2G^2]^{-1}$$

in the following regimes ($\eta \equiv m_B/m_F$):

$$\begin{aligned} &\text{either} \quad \eta > 1,\ e_{1,3}^2 < 16\pi m_F\{\mu(\eta-1)[(\eta+1)\eta^{-1}f(\eta^{-2})-1]\}^{-1}, \\ &\text{or} \quad \eta < 1,\ e_{1,3}^2 < 16\pi m_F\eta^2\{\mu(1-\eta)[1-(1-\eta)f(\eta^2)]\}^{-1}. \end{aligned} \tag{16}$$

The corresponding squared masses (poles of (11c, f)) are obtained as solutions of the transcendental equations:

$$\begin{aligned} &2x^{1/2}f(x) = (1+x)f(x) + 16\pi m_F(e_{1,3}^2\mu)^{-1}x - 1 + \eta - \eta(1-x/\eta^2)f(x/\eta^2), \\ &x \equiv m_{A(0),B}^2/4m_F^2,\ m_{A(0),B} < \min\{2m_F, 2m_B\},\ e_{1,3}^2,\ \eta \text{ satisfying (16).} \end{aligned} \tag{17}$$

Let us stress that the appearance of (15) in (11c, f) can be viewed as a result of dynamical generation in the effective $1/N$ action S_1 (6) of explicit TGIMTs [1, 2] (in the low-energy limit: $|p|^2 \ll m_{B,F}^2$):

$$S_{\mathrm{TGIMT}} = \frac{Nn}{16\pi}\int d^3x\epsilon_{\mu\nu\lambda}\{A_{(0)}^{\mu}F^{\nu\lambda}(A_{(0)}) + \mathrm{tr}\,(\underset{\sim}{A}^{\mu}\underset{\sim}{F}^{\nu\lambda}(\underset{\sim}{A}) - i\tfrac{2}{3}\underset{\sim}{A}^{\mu}\underset{\sim}{A}^{\nu}\underset{\sim}{A}^{\lambda}). \tag{15'}$$

(β) Three SU(n)-singlets and three adjoint-SU(n) multiplets of massive *composite* Majorana fermions, whose fields $\Phi_{(0),A}^i$, $i = 1, 2, 3$, are the combinations of $\kappa_{(0),A}$, $C_{(0),A}$, $\rho_{(0),A}$, resp., diagonalizing the mixed propagators (12), in the following regimes ($\eta' \equiv (m_B + m_F)(2m_F)^{-1}$):

$$\begin{aligned} &\eta' > 1 \quad \text{for } \Phi_{(0),A}^1 \\ &\eta' > 1, \quad e_{2,4}^2 < 16u_{4,8}\eta' m_F/\mu \quad \text{for } \Phi_{(0),A}^{2,3}. \end{aligned} \tag{18}$$

The corresponding squared masses (poles of the eigenvalues of the diagonalized matrix of κ, C, ρ-propagators (12)) are obtained as solutions of the transcendental equations:

$$\begin{aligned} &x = [1 + x^{-1/2}J_1(x) - \pi\eta'(u_{2,6} + 1/u_{4,8})(4f(x/\eta'^2))^{-1}]^2 \cdot [1 + K_1(x)]^{-2}, \\ &x \equiv m_{\Phi_{(0),A}^1}^2/4m_F^2 < \eta'^2, \quad \eta' > 1; \end{aligned} \tag{19}$$

$$\begin{aligned} &x_{(\pm)} = [1 \pm x_{\pm}^{-1/2}J_2(x_{(\pm)}) - \pi\eta'(4u_{4,8}f(x_{(\pm)}/\eta'^2))^{-1}]^2 \times \\ &\quad \times [1 \pm K_2(x_{(\pm)}) + 4\pi\eta' m_F(\mu e_{2,4}^2 f(x_{(\pm)}/\eta'^2))^{-1}]^{-2}, \end{aligned} \tag{20}$$

$$x_{(+)} \equiv m_{\Phi_{(0),A}^2}^2/4m_F^2, \quad x_{(-)} \equiv m_{\Phi_{(0),A}^3}^2/4m_F^2,\ \eta',\ e_{2,4}^2 \text{ satisfying (18),}$$

where J_i, K_i, $i = 1, 2$, are defined as follows:

$$J_1(x) \pm K_1(x) = [\pm 2x^{1/2} + 1 + x + \pi^2 \eta'^2 (u_{2,6} - 1/u_{4,8})^2 (4f(x/\eta'^2))^{-2}]^{1/2},$$

$$J_2(x) \pm K_2(x) = \{\pm 2x^{1/2} [1 + \pi^2 \eta'^2 m_F (u_{4,8} e^2_{2,4} \mu f^2 (x/\eta'^2))^{-1}] + \qquad (21)$$

$$+ 1 + \pi^2 \eta'^2 (4u_{4,8} f(x/\eta'^2))^{-2} + x[1 + 16\pi^2 \eta'^2 m_F^2 (\mu e^2_{2,4} f(x/\eta'^2))^{-2}]\}^{1/2}.$$

Recall that on a classical level according to (2):

$$\kappa_{(0)} = 8(Nnu_2)^{-1}(\varphi^* \psi + \overline{\psi} \mathscr{C} \varphi), \qquad \kappa_A = 8(Nnu_6)^{-1}(\varphi^* \tau_A \psi + \overline{\psi} \mathscr{C} \tau_A \varphi),$$

$$\rho_{(0)} = 16u_4 (Nn)^{-1} \varphi^* \psi, \qquad \rho_A = 16u_8 (Nn)^{-1} \varphi^* \tau_A \psi,$$

the classical $C_{(0),A}$-fields are massless, and in the limiting case (3):

$$\kappa_{(0)} + \frac{i}{2} C_{(0)} = -T(Nn\mu)^{-1}(\varphi^* i \not\partial \psi), \kappa_A + \frac{i}{2} C_A = -T(Nn\mu)^{-1}(\varphi^* \tau_A i \not\partial \psi).$$

(γ) Adjoint SU(n) multiplet of massive *composite* bosons σ_A;

$$\sigma_A = 8(Nnu_5)^{-1} \varphi^* \tau_A \varphi \text{ (for (2))}, \ \sigma_A = (2Nn\mu a_3)^{-1} \overline{\psi} \tau_A \psi \text{ (for (3))},$$

in the regime:

$$\eta > 1, \ [1 + u_5^2 \pi^2 \eta (4f(\eta^{-2}))^{-1} - \tfrac{1}{2}\pi u_9] \frac{m_F}{4\pi\mu} < 1/T_c - a_3 < [2 + \tfrac{1}{4} u_5^2 \pi^2 \eta - \tfrac{1}{2} \pi u_9] \frac{m_F}{4\pi\mu}, \qquad (22)$$

with a mass squared obtained as a solution of the transcendental equation:

$$(1 - x) f(x) - \frac{4\pi\mu}{m_F} (1/T_c - a_3) + 1 - \tfrac{1}{2}\pi u_9 + u_5^2 \pi^2 \eta (4f(x/\eta^2))^{-1} = 0, \qquad (23)$$

$$x \equiv m^2_{\sigma_A} / 4m_F^2 < 1, \quad a_3 \text{ satisfying (22)}.$$

The existence and uniqueness of the solutions to (17), (19), (20), (23) follow directly by using the explicit form of $f(z)$ (11e) and (21).

Higher orders in $1/N$ will in general shift the values of all masses, however, the structure of the particle spectra remains valid, provided the $1/N$ expansion of HMF_3 were true as an asymptotic series. To prove the latter, however, is a highly non-trivial task lying beyond the scope of the present note*.

Finally, let us stress that the generalized non-linear sigma models with fermions (3) are the

---

*Asymptotic convergence of $1/N$ expansion was proved so far only for some much more simple models [9].

simplest members of the class HMF_3 (1) which exhibit both interesting features considered above: dynamical generation of TGIMTs (*without P, T-breaking* on classical level), composite fermions $\Phi_{(0),A}$ ($\Phi_{(0),A} = \kappa_{(0),A} + (i/2)C_{(0),A}$, $m_\Phi = 2m_F$ ($m_B > m_F$) in this case, cf. (3) and (12)) and composite bosons σ_A (two-fermion bound states) (for details, see [10]).

REFERENCES

1. Siegel, W., *Nucl. Phys.* **B156**, 135 (1979);
   Jackiw, R. and Templeton, S., *Phys. Rev.* **D23**, 2291 (1981);
   Schonfeld, J., *Nucl. Phys.* **B185**, 157 (1981).
2. Deser, S., Jackiw, R., and Templeton, S., *Ann. Phys.* (1982) (to appear);
   Jackiw, R., in *F.E. Low Festschrift*, MIT Press, Cambridge, MA, 1982.
3. Nissimov, E.R. and Pacheva, S.J., *Lett. Math. Phys.* **5**, 67, 333 (1981).
4. Nissimov, E.R. and Pacheva, S.J., *Lett. Math. Phys.* **6**, 101 (1982).
5. Witten, E., *Nucl. Phys.* **B149**, 285 (1979);
   D'Adda, A., Lüscher, M., and Di Vecchia, P., *Nucl. Phys.* **B152**, 125 (1979).
6. Brezin, E., Le Guillou, J., and Zinn-Justin, J., in C. Domb and M.S. Green (eds.) *Phase Transitions and Critical Phenomena*, vol. 6, Academic Press, N.Y., 1976.
7. Lowenstein, J. and Zimmermann, W., *Comm. Math. Phys.* **46**, 105 (1976);
   Lowenstein, J., *Comm. Math. Phys.* **47**, 53 (1976).
8. Nissimov, E.R. and Pacheva, S.J., to be submitted to *Lett. Math. Phys.*
9. Kupiainen, A., *Comm. Math. Phys.* **73**, 273; **74**, 199 (1980).
10. Nissimov, E.R. and Pacheva, S.J., in A.N. Tavkhelidze *et al.* (eds.) *Proc. of the International Seminar 'Quarks-82'* (Sukhumi, May 5-7, 1982) (to appear).

(Received May 7, 1982)